University of Diyala
College of Engineering
Department of Materials

Fundamentals of Electric Circuits

Lecture Four

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2-2 Effect of Temperature on Resistance

The effect of rise in temperature is:

(*i) to increase the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature.*

(ii) to increase the resistance of alloys, though in their case, the increase is relatively small and irregular.

(iii) to decrease the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon.

2-3 Temperature Coefficient of Resistance

Let a metallic conductor having a resistance of R_o at 0°C be heated of t°C and let its resistance at this temperature be R_t. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

(*i) directly on its initial resistance*

(ii) directly on the rise in temperature

(iii) on the nature of the material of the conductor.

Or $\boldsymbol{R_t - R_0 \propto R \times t}$ ***or*** $\boldsymbol{R_t - R_0 = \alpha\, R_0\, t}$ $\qquad (i)$

Where α (alpha) is a constant and is known as the temperature coefficient of resistance of the conductor.

Rearranging Eq. *(i)*, we $get\, \alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\Delta R}{R_0 \times t}$

If $R_0 = 1\Omega, t = 1°C, then\ \alpha = \Delta R = R_t - R_0$

Hence, the temperature-coefficient of a material may be defined as:

The increase in resistance per ohm original resistance per °C rise in temperature.

From Eq. *(i)*, we find that $\boldsymbol{R_t = R_0(1 + \alpha\, t)}$ ... $(\boldsymbol{ii})$

In Fig. 2.2 is shown the temperature/resistance graph for copper and is practically a straight line. If this line is extended backwards, it would cut the temperature axis at a point where temperature is − 234.5°C (a number quite easy to remember). It means that theoretically, the resistance of copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 2.2 it is seen that:

$$\frac{R_t}{R_0} = \frac{t + 234.5}{234.5} = \left(1 + \frac{t}{234.5}\right)$$

$$R_t = R_0\left(1 + \frac{t}{234.5}\right) \; or \; R_t = R_0(1 + \alpha t) \; where \; \alpha = \frac{1}{234.5} \; for \; copper$$

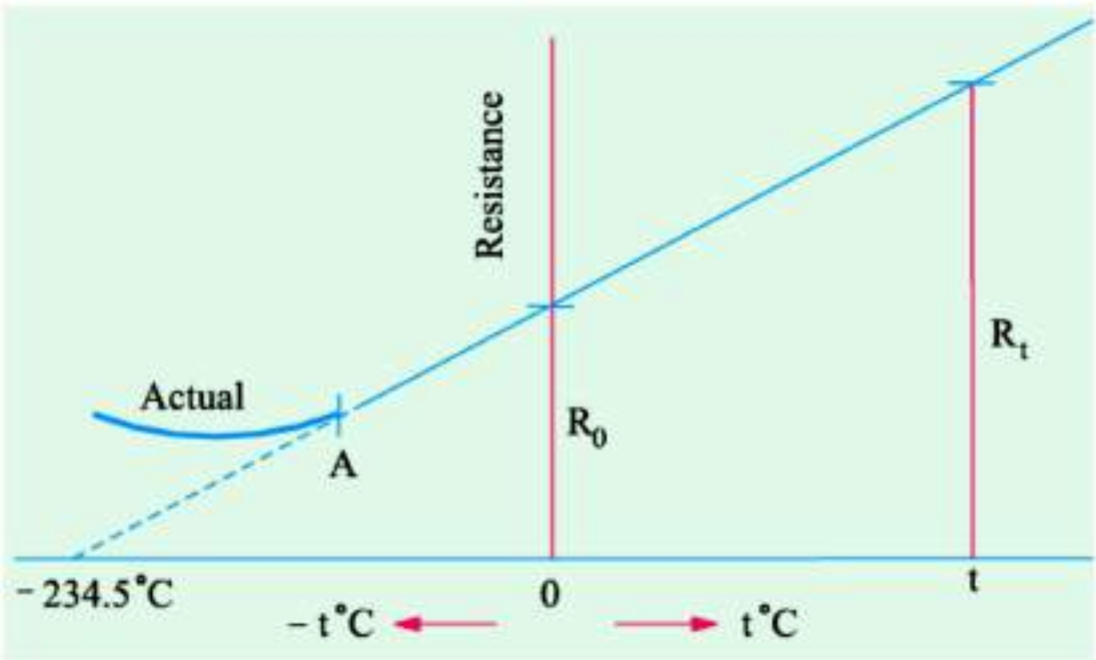

Fig. 2.2

2-4 Value of α at Different Temperatures

So far we did not make any distinction between values of α at different temperatures. But it is found that value of α itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then α has the value of α_0. At any other initial temperature t°C, value of α is α_t and so on. It should be remembered that, for any conductor, α_0 has the maximum value.

Suppose a conductor of resistance R_0 at 0°C (point A in Fig. 2.3) is heated to t°C (point B). Its resistance R_t after heating is given by

$$\boldsymbol{R_t = R_0(1 + \alpha_0\, t)} \qquad (\boldsymbol{i})$$

Where α_0 is the temperature-coefficient at 0°C.

Now, suppose that we have a conductor of resistance R_t at temperature t°C. Let this conductor be cooled from t°C to 0°C. Obviously, now the initial point is B and the final point is A. The final resistance R_0 is given in terms of the initial resistance by the following equation

$$\boldsymbol{R_0 = R_t\,[1 + \alpha_t(-t)] = R_t(1 - \alpha_t\, t)} \qquad (\boldsymbol{ii})$$

From Eq. *(ii)* above, we have $\alpha_t = \frac{R_t - R_0}{R_t \times t}$

Substituting the value of R_t from Eq. *(i)*, we get

$$\boldsymbol{\alpha_t = \frac{R_0(1+\alpha_0 t) - R_0}{R_0(1+\alpha_0 t)\times t} = \frac{\alpha_0}{1+\alpha_0 t}} \qquad \therefore\ \boldsymbol{\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}} \qquad \ldots(\boldsymbol{iii})$$

In general, let α_1 = tempt. coeff. at t_1°C ; α_2 = tempt. coeff. at t_2°C. Then from Eq. *(iii)* above, we get

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0\, t_1} \ or\ \frac{1}{\alpha_1} = \frac{1 + \alpha_0\, t_1}{\alpha_0}$$

Similarly,

$$\frac{1}{\alpha_2} = \frac{1 + \alpha_0\, t_2}{\alpha_0}$$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) or\ \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) or\ \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

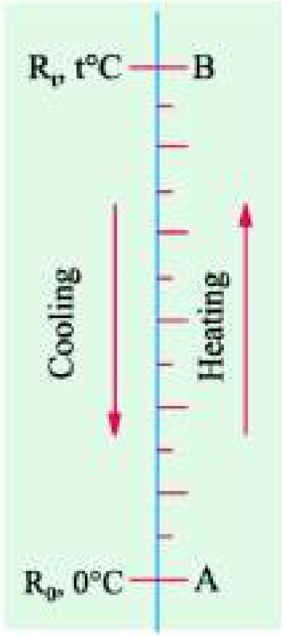

Fig. 2.3

Values of α for copper at different temperatures are given in Table No. 2.2.

Tempt. in °C	0	5	10	20	30	40	50
α	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

Table 2.2. Different values of α for copper

In view of the dependence of α on the initial temperature, we may define ***the temperature coefficient of resistance at a given temperature as the charge in resistance per ohm per degree centigrade change in temperature from the given temperature.***

In case R_0 is not given, the relation between the known resistance R_1 at t_1°C and the unknown resistance R_2 at t_2°C can be found as follows:

$$R_2 = R_0(1 + \alpha_0 t_2) \; and \; R_1 = R_0(1 + \alpha_0 t_1)$$

$$\boldsymbol{\frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1}} \qquad (iv)$$

Or $\boldsymbol{R_2 = R_1[1 + \alpha_0(t_2 - t_1)]}$

2-5 Variations of Resistivity with Temperature

Not only resistance but specific resistance or resistivity of metallic conductors also increases with rise in temperature.

As seen from Fig. 2.4 the resistivities of metals vary linearly with temperature over a significant range of temperature-the variation becoming non-linear both at very high and at very low temperatures. Let, for any metallic conductor,

ρ_1 = resistivity at t_1°C

ρ_2 = resistivity at t_2°C

m =Slope of the linear part of the curve

Then, it is seen that

$$m = \frac{\rho_2 - \rho_1}{t_2 - t_1}$$

Or $\rho_2 = \rho_1 + m(t_2 - t_1)$ or $\frac{\rho_2}{\rho_1} = 1 + \frac{m}{\rho_1}(t_2 - t_1)$

The ratio of m/ρ_1 is called the *temperature coefficient of resistivity* at temperaturet_1°C. It may be defined as numerically equal to the fractional change in ρ_1 per °C change in the temperature from t_1°C. It is almost equal to the temperature-coefficient of resistance α_1. Hence, putting $\alpha_1 = m/\rho_1$, we get

$$\boldsymbol{\rho_2 = \rho_1[1 + \alpha_1(t_2 - t_1)\ or\ simply\ as\ \rho_t = \rho_0(1 + \alpha_0 t)}$$

Example 1. *A copper conductor has its specific resistance of* 1.6×10^{-6} *ohm-cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find (i) the specific resistance and (ii) the resistance - temperature coefficient at 60°C.*

Solution.

$$R_{100} = R_0\,(1 + 100\,\alpha_0) \qquad ...(i)$$

$$R_{40} = R_0\,(1 + 40\,\alpha_0) \qquad ...(ii)$$

$$\therefore \quad \frac{3.767}{3.146} = \frac{1 + 100\,\alpha_0}{1 + 40\,\alpha_0} \quad \text{or} \quad \alpha_0 = 0.00379 \quad \text{or} \quad 1/264 \text{ per°C}$$

From (*i*), we have

$$3.767 = R_0\,(1 + 100 \times 0.00379) \quad \therefore \; R_0 = \mathbf{2.732\ \Omega}$$

Now,

$$\alpha_{40} = \frac{\alpha_0}{1 + 40\,\alpha_0} = \frac{0.00379}{1 + 40 \times 0.00379} = \frac{\mathbf{1}}{\mathbf{304}} \textbf{ per°C}$$

Example 2. *A platinum coil has a resistance of 3.146 Ω at 40°C and 3.767 Ω at 100°C. Find the resistance at 0°C and the temperature-coefficient of resistance at 40°C.*

Solution.

$$R_{100} = R_0\,(1 + 100\,\alpha_0) \qquad ...(i)$$

$$R_{40} = R_0\,(1 + 40\,\alpha_0) \qquad ...(ii)$$

$$\therefore \quad \frac{3.767}{3.146} = \frac{1 + 100\,\alpha_0}{1 + 40\,\alpha_0} \quad \text{or} \quad \alpha_0 = 0.00379 \quad \text{or} \quad 1/264 \text{ per°C}$$

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Example 3. *A potential difference of 250 V is applied to a field winding at 15°C and the current is 5 A. What will be the mean temperature of the winding when current has fallen to 3.91 A, applied voltage being constant. Assume* $\alpha_{15} = 1/254.5$.

Solution. Let R_1 = winding resistance at 15°C; R_2 = winding resistance at unknown mean temperature t_2°C.

$$\therefore \quad R_1 = 250/5 = 50\ \Omega \quad R_2 = 250/3.91 = 63.94\ \Omega$$

Now

$$R_2 = R_1\,[1 + \alpha_{15}\,(t_2 - t_1)] \quad \therefore \; 63.94 = 50\left[1 + \frac{1}{254.5}(t_2 - 15)\right]$$

$$\therefore \quad t_2 = \mathbf{86°C}$$

Example 4. *Two coils connected in series have resistances of 600 Ω and 300 Ω with tempt. coeff. of 0.1% and 0.4% respectively at 20°C. Find the resistance of the combination at a tempt. Of 50°C. What is the effective tempt. coeff. of combination ?*

Solution. Resistance of 600 Ω resistor at 50°C is = 600 [1 + 0.001 (50 − 20)] = 618 Ω

Similarly, resistance of 300 Ω resistor at 50°C is = 300 [1 + 0.004 (50 − 20)] = 336 Ω

Hence, total resistance of combination at 50°C is = 618 + 336 = **954 Ω**

Let β = resistance-temperature coefficient at 20°C

Now, combination resistance at 20°C = 900 Ω

Combination resistance at 50°C = 954 Ω

$$\therefore \quad 954 = 900\,[1 + \beta\,(50 - 20)] \quad \therefore\ \beta = \mathbf{0.002}$$

Example 5. *The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20°C of 4.3 μΩ-cm. If α = 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 watts at a filament tempt. of 2420°C ?*

Solution. Electric power generated = $I^2 R$ watts = V^2/R watts

$$\therefore \quad V^2/R = 60 \quad \text{or} \quad 240^2/R = 60$$

Resistance at 2420°C $\quad R_{2420} = \dfrac{240 \times 240}{60} = 960\,\Omega$

Now $\quad R_{2420} = R_{20}\,[1 + (2420 - 20) \times 0.005]$

or $\quad 960 = R_{20}\,(1 + 12)$

$$\therefore \quad R_{20} = 960/13\ \Omega$$

Now $\quad \rho_{20} = 4.3 \times 10^{-6}\ \Omega\text{-cm} \quad \text{and} \quad A = \dfrac{\pi(0.002)^2}{4}\ \text{cm}^2$

$$\therefore \quad l = \frac{A \times R_{20}}{\rho_{20}} = \frac{\pi\,(0.002)^2 \times 960}{4 \times 13 \times 4.3 \times 10^{-6}} = \mathbf{54\ cm}$$

Example 6. *A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of Cu at 20°C = 1.724 × 10−6 ohm*-cm *and resistance tempt. coeff. of Cu at 0°C = 0.0043/°C.*

Solution. The semi-circular ring is shown in Fig. 1.11.

Mean radius of ring $= (6 + 9)/2 = 7.5$ cm

Mean length between end faces $= 7.5\,\pi$ cm $= 23.56$ cm

Cross-section of the ring $= 3 \times 4 = 12\ \text{cm}^2$

Now $\alpha_0 = 0.0043/°\text{C}$; $\alpha_{20} = \dfrac{0.0043}{1+20\times 0.0043} = 0.00396$

$$\rho_{50} = \rho_{20}[1 + \alpha_0(50-20)]$$
$$= 1.724\times 10^{-6}(1+30\times 0.00396) = 1.93\times 10^{-6}\ \Omega\text{-cm}$$

$$R_{50} = \frac{\rho_{50}\times l}{A} = \frac{1.93\times 10^{-6}\times 23.56}{12} = \mathbf{3.79\times 10^{-6}\ \Omega}$$

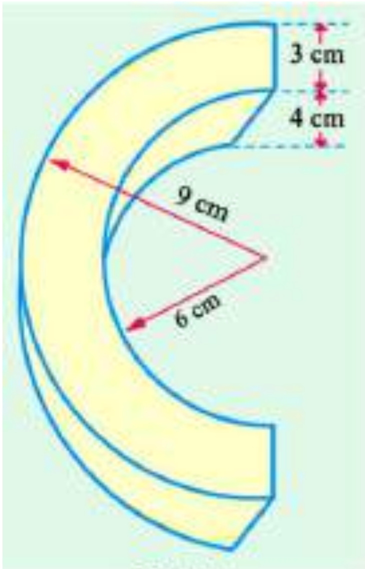

Fig.1.11